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Retail-Chain Multi-Item Optimization: A Mixed Integer Non-Linear (MINL) Heuristic Approach

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Abstract

The model studies the optimization of mixed integer non-linear retail-chain multi-item model. In this model publicity and no items lost due to deterioration are the significant factors with the decision parameters are retailer's order quantity publicity effort factor for multi-item and cycle time. Retailer's order quantity for deteriorated multi-item is an integer and the publicity effort for each multi-item may be integer or fraction. So a heuristic approach is applied in a mixed integer non-linear retail-chain multi-item model. The market demand may increase with the publicity of the multi-item over time when the units do not lost due to deterioration. In this model, publicity effort and replenishment decision are adjusted arbitrarily upward or downward for profit maximization model in response to the change in market demand within the planning horizon. The numerical analysis and comparative analysis show that an appropriate publicity policy can benefit the retailer. Finally, sensitivity analysis of the optimal solution with respect to the major inventory parameters is also studied to draw the managerial implications with retailer's perspective.

Keywords: Retail-chain, Multi-item, Order quantity, Mixed integer non-linear programming (MINLP), Optimization

Introduction

Inventory management plays a significant role in businesses since it can help the retail companies to reach the goal ensuring prompt delivery, avoiding shortages, helping sales at competitive prices with cost effective and efficiency. Inventory management is one of the most important components of the production function where the production function is the mid between procurement function and physical distribution function. Since inventory management is the key factor of the production function so, the success of inventory management is the key to the success of production system in a company. The mission of inventory management are the right quality, the right quantity, the right condition, the right cost tradeoffs, the right customer, the right information, the right response, the right time, the right value, the right product and the right place. The mathematical modeling of real-world inventory problems necessitates the simplification of assumptions to make the mathematics flexible. However, excessive simplification of assumptions results in mathematical models that do not represent the inventory situation to be analyzed. Many models have been proposed to deal with a variety of inventory problems. The classical analysis of single-product inventory control considers three costs for holding inventories. These costs are the ordering cost, carrying cost and shortage cost.

The classical analysis builds a model of an inventory system and calculates the EOQ of single-product inventory model which minimize these three costs so that their sum is satisfying minimization criterion. One of the unrealistic assumptions is that items stocked preserve their physical characteristics during their stay in inventory. Items in stock are subject to many possible risks, e.g. damage, spoilage, dryness; vaporization etc., those results decrease of usefulness of the original one and a cost is incurred to account for such risks. To control an inventory system, one cannot be ignored demand since inventory is partially determined by demand, as suggested by Waters (1994) and Osteryoung, Mc Carty and Reinhart (1986) in many cases a small change in the demand pattern may result in a large change in optimal inventory decisions. A manager of a company has to investigate the factors that influence demand pattern, because customers' purchasing behaviour may be affected by factors such as promotional effort, units lost due to deterioration, quantity ordered, profit and so on. A subject in the area of inventory theory that has recently been receiving considerable attention is the class of inventory models with deterioration. With these models, the presence of retail inventory is assumed to have a motivating effect on the customer.

Many models have been proposed to deal with a variety of inventory problems. Comprehensive reviews of inventory models can be found in Rafaat (1991) and Jain and Silver (1994). In deterministic inventory models, many are developed under the assumption that demand is either constant or stock dependent for deteriorated items. Bose, Goswami and Chaudhuri (1995) dealt with the EOQ problem for deteriorating items with linear time dependent demand rate under inflation where shortages and discounts are allowed. Goyal and Gunasekaran (1995) considered the effect of different market policies, e.g. the price per product and advertisement frequency on the demand of a perishable item. Gupta and Gerchak (1995) analyzed two scenarios; the first considers TOD as a constant and the store manager may choose an appropriate value, while the second assumes that TOD is a random variable. Hariga (1995) proposed the correct theory for the problem supplied with numerical examples. The most recent work found in the literature is that of Hariga (1996) who extended his earlier work by assuming a time-varying demand over a finite planning horizon. Padmabhan and Vrat (1995) presented an EOQ inventory model for perishable items with a stock dependent selling rate. Unlike the work of Wee (1993) who studied the case of partial backlogging for deteriorating items, Salameh, Jaber and Noueihed (1999) studied an EOQ inventory model in which it assumes that the percentage of on-hand inventory wasted due to deterioration is a characteristic feature of the inventory conditions which govern the item stocked.

Nowadays, retailer publicity activity has become more and more common in real business world. For example, Wal-Mart, Costco, Reliance Fresh, Big Bazar and The World often try to stimulate demand for specific types of electric equipment by offering price discounts; clothiers Baleno, NET, Reliance Trends make shelf space for specific clothes items available for longer periods; McDonald's, Burger King, Pizza Hut and KFC often use coupons to attract consumers. Other publicity strategies include free goods, advertising, displays and so on. The publicity policy is very important for the retailer. How much publicity effort the retailer makes has a big impact on annual profit. Residual costs may be incurred by too much publicity while too few may result in lower sales revenue. The prices of multi-items, for example FMCG products, automotive parts, electronic parts etc. will be shipped at a time to the super market or retail outlet. Publicity activities, such as price promotion and advertisements, are commonly employed to speed up the movement of the products. The making of joint optimal retail price and publicity effort decisions while considering demand in multi-item industry is therefore important. The model with publicity effort factor provides the decision maker with useful and practical insights. Tsao and Sheen (2008) discussed dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment. This study addresses the problem by proposing a multi-echelon supply chain model with publicity. So optimization of the proposed model has been studied by adding publicity effort cost with publicity constraint. The objective is to determine the retailer's optimal publicity effort and replenishment policies for multi-item. Hariga (1994) studied the effects of inflation and time value of money on the replenishment policies of items with time continuous non-stationary demand over a finite planning horizon.

This study addresses the problem by proposing a continuous review inventory model under publicity by assuming that the units do not lost due to deterioration of the items. In this model optimization has been studied for applying publicity effort cost with publicity constraint. The effect of deteriorated multi-item on the instantaneous profit maximization replenishment model under publicity is considered in this model. The market demand may increase with the publicity of the product over time when the units do not lost due to deterioration. In the existing literature about promotion it is assumed that the promotional effort cost is a function of promotion. Tripathy and Pattnaik (2008, 2011) study profit maximization entropic order quantity model for deteriorated items with stock dependent demand where discounts are allowed for acquiring more profit. In this model, promotional effort and replenishment decision are adjusted arbitrarily upward or downward for profit maximization model in response to the change in market demand within the planning horizon. Tsao and Sheen (2012) developed models and algorithms for solving multi-item replenishment problems for different constraints. Hence the multi-item optimization integer EOQ model is the focus of the present study.

The objective of this model is to determine optimal publicity efforts and integer replenishment quantities in an instantaneous replenishment profit maximization multi-item model.

All mentioned above inventory literatures with deterioration or no wasting the percentage of on-hand inventory due to deterioration have the basic assumptions that the retailer owns a storage room with optimal order quantity. In recent years, companies have started to recognize that a trade-off exists between product varieties in terms of quality of the product for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality of the product, these companies have mainly relied on qualitative judgment. This multi-item model postulates that measuring the behaviour of production systems may be achievable by incorporating the idea of retailer in making optimum decision on replenishment of traditional model. The major assumptions used in the above research articles are summarized in Table 1.

Table 1: Summary of the Related Researches

Author(s) and Published Year	Programming	Structure of the Model	Item	Demand	Demand Patterns	Deterioration	Publicity Effort Cost	Planning	Units Lost due Deteri oration	Model
Hariga (1994)	Non-linear	Crisp (EOQ)	Single	Time	Non-stationary	Yes	No	Finite	No	Cost
Salameh et al. (1999)	Non-linear	Crisp (EOQ)	Single	Constant (Determin istic)	Constant	Yes	No	Finite	Yes	Cost
Tsao et al. (2008)	Non-linear	Crisp (EOQ)	Single	Time and Price	Linear and Decreasi ng	Yes	Yes	Finite	No	Profit
Tsao et al. (2012)	Non-linear	Multi-echelon Supply chain	Multi	Price	Linear Decreasi ng	No	No	Finite	No	Profit
Present Model (2018)	Mixed Integer Non-linear	Multi-item Retail-chain (EOQ)	Multi	Constant (Determin istic)	Constant	Yes (Wasting)	Yes	Finite	No	Profit

The remainder of the model is organized as follows. In Section 2 assumptions and notations are provided for the development of the model. The mathematical formulation is developed in Section 3. Algorithm through steps is outlined in Section 4 to obtain the best solution for the mixed integer multi-item model. The solution procedure is given in Section 5. In Section 6, numerical example with comparative analysis is presented to illustrate the development of the model. The sensitivity analysis is carried out in Section 7 to observe the changes in the optimal solution. Finally in Section 8 the summary and the concluding remarks are explained.

Assumptions and Notations

The following notations are used:

k Number of items considered,

r_i Consumption rate for item i,

t_{ci} Cycle length for item i,

t_c Cycle length, $t_c = \sum_{i=1}^{k} t_{ci}$

t_c^* Optimal cycle length, $t_c^* = \sum_{i=1}^{k} t_{ci}^*$

c_i Purchasing cost for item i,

p_i Selling Price for item i,

α_i Percentage of on-hand inventory of item i that is lost due to deterioration, $\alpha_i \to 0$

q_i Integer ordering quantity for item i, $q = \sum_{i=1}^{k} q_i$

q_i^* Optimal integer ordering quantity for item i, $q^* = \sum_{i=1}^{k} q_i^*$

ρ_i The publicity effort factor per cycle,

PEC (ρ_i) The publicity effort cost, $PE\ (\rho_i) = \tau_i(\rho_i - 1)^2 {r_i}^{\beta_i}$ where, $\tau_i > 0$ and β_i is a constant,

h_i Holding cost of item i per unit per unit of time,

HC (q_i, ρ_i) Holding cost per cycle,

A Major ordering cost per order,

a_i Minor ordering cost for item i

$\varphi(t_i)$ On-hand inventory level at time t_i of item i,

$\pi_1(q_i, \rho_i)$ Net profit per cycle

$\pi\ (q_i, \rho_i)$ Average profit per cycle, $\pi\ (q_i, \rho_i) = \frac{\pi_1(q_i)}{t_c}$

$\pi_1^*(q_i, \rho_i)$ Optimal net profit per cycle

$\pi^*(q_i, \rho_i)$ Optimal average profit per cycle

Mathematical Model

Denote $\varphi(t_i)$ as the on-hand inventory level at time t_i, i=1,2,............k. During a change in time from point t_i to $t_i + \mathrm{d}t_i$, where $t_i + \mathrm{d}t_i > t_i$, the on-hand inventory drops from $\varphi(t_i)$ to $\varphi(t_i + \mathrm{d}t_i)$. Then $\varphi(t_i + \mathrm{d}t_i)$ is:

$$\varphi(t_i + \mathrm{d}t_i) = \varphi(t_i) - r_i\rho_i dt_i - \alpha_i\varphi(t_i)dt_i, i = 1,2, \dots \dots \dots \dots \dots k$$

$\varphi(t_i + \mathrm{d}t_i)$ can be re-written as: $\frac{\varphi(t_i+\mathrm{d}t_i)-\varphi(t_i)}{dt_i} = -r_i\rho_i - \alpha_i\varphi(t_i)$ and $dt_i \to 0$, equation $\frac{\varphi(t_i+\mathrm{d}t_i)-\varphi(t_i)}{dt_i}$ reduces to: $\frac{d\varphi(t_i)}{dt_i} + \alpha_i\varphi(t_i) + r_i\rho_i = 0$. It is a differential equation, solution is $\varphi(\mathrm{t_i}) = \frac{-\mathrm{r_i}\rho_i}{\alpha_\mathrm{i}} + \left(\mathrm{q_i} + \frac{\mathrm{r_i}\rho_i}{\alpha_\mathrm{i}}\right) \times \mathrm{e}^{-\alpha_\mathrm{i}\mathrm{t_i}}$

Where $\mathrm{q_i}$is the order quantity which is instantaneously replenished at the beginning of each cycle of length $\mathrm{t_{ci}}$ units of time. The stock is replenished by $\mathrm{q_i}$ units each time these units are totally depleted as a result of outside demand and deterioration. Behaviour of the inventory level for the above model is illustrated in Fig. 1.The cycle length, $\mathrm{t_{ci}}$ is determined by first substituting $\mathrm{t_{ci}}$ into equation $\varphi(t_i)$ and then setting it equal to zero to get: $\mathrm{t_c} = \sum_{\mathrm{i}=1}^{\mathrm{k}} \mathrm{t_{ci}} = \sum_{\mathrm{i}=1}^{\mathrm{k}} \frac{1}{\alpha_\mathrm{i}} \ln\left(\frac{\alpha_\mathrm{i}\mathrm{q_i} + \mathrm{r_i}\rho_i}{\mathrm{r_i}\rho_i}\right)$.

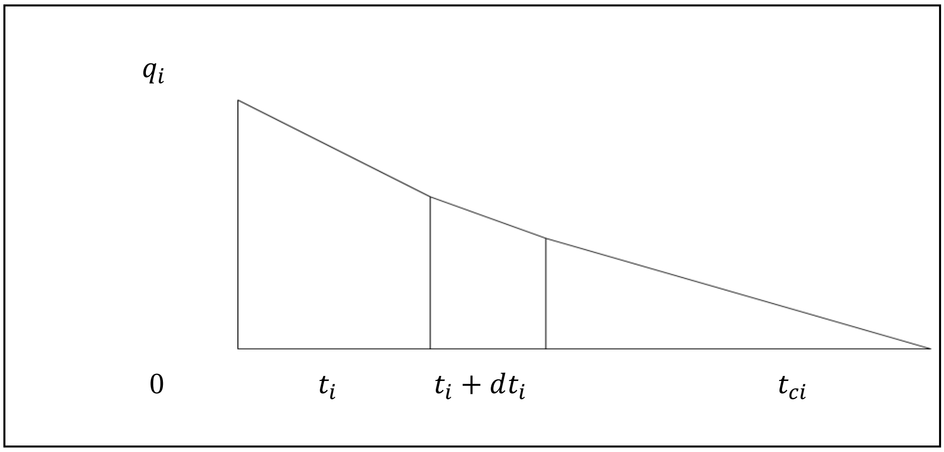


Figure 1: Behaviour of the inventory over a cycle for a deteriorating item

Equation $\varphi(t_i)$ and t_{ci} are used to develop the mathematical model. Then the total number of units lost per cycle, L, is given as: $L = \sum_{i=1}^{k} L_i = \sum_{i=1}^{k} r_i\rho_i \left[\frac{q_i}{r_i\rho_i} - \frac{1}{\alpha_i} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right]$. It is worthy to mention that as $\alpha_i$approaches to zero, t_{ci} approaches to $\frac{q_i}{r_i\rho_i}$ and $t_c = \sum_{i=1}^{k} t_{ci} = \sum_{i=1}^{k} \frac{q_i}{r_i\rho_i}$ and L, the number of units lost per cycle due to deterioration is zero.

The total cost per cycle, $TC(q_i, \rho_i)$, is the sum of the major ordering cost per order, minor ordering cost, the holding cost per cycle, purchasing cost per cycle and publicity effort cost per cycle.

HC (q_i, ρ_i) is obtained from equation $\varphi(t_i)$ as:

$$HC = \sum_{i=1}^{k} \int_0^{t_{ci}} h_i \varphi(t_i) dt_i = \sum_{i=1}^{k} h_i \int_0^{\frac{1}{\alpha_i} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)} \left[-\frac{r_i\rho_i}{\alpha_i} + \left(q_i + \frac{r_i\rho_i}{\alpha_i}\right) \times e^{-\alpha_i t_i}\right] dt_i$$

So, $HC = \sum_{i=1}^{k} h_i \left[\frac{q_i}{\alpha_i} - \frac{r_i\rho_i}{\alpha_i^2} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right]$

$$TC(q_i, \rho_i) = Major\ Ordering\ Cost\ (MAOC) + Minor\ Ordering\ Cost\ (MOC) + Holding\ Cost\ (HC) + Purchasing\ Cost\ (PC) + Publicity\ Effort\ Cost\ (PEC)$$

$$TC(q_i, \rho_i) = A + \sum_{i=1}^{k} \left[a_i + h_i \left[\frac{q_i}{\alpha_i} - \frac{r_i\rho_i}{\alpha_i^2} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right] + c_i q_i + \tau_i(\rho_i - 1)^2 {r_i}^{\beta_i}\right]$$

Where, $MAOC = A,\ MOC = \sum_{i=1}^{k} a_i$, $HC = \sum_{i=1}^{k} h_i \left[\frac{q_i}{\alpha_i} - \frac{r_i\rho_i}{\alpha_i^2} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right]$, $PC = \sum_{i=1}^{k} c_i q_i\ and\ PEC = \sum_{i=1}^{k} \tau_i(\rho_i - 1)^2 {r_i}^{\beta_i}$

The total cost per unit of time, $TCU(q_i, \rho_i)$, is given by dividing equation $TC(q_i, \rho_i)$ by t_c to give:

$$TCU(q_i, \rho_i) = TC(q_i, \rho_i) \times \sum_{i=1}^{k} \left[\frac{1}{\alpha_i} \ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right]^{-1} = \left[A + \sum_{i=1}^{k} \left[a_i + h_i \left[\frac{q_i}{\alpha_i} - \frac{r_i\rho_i}{\alpha_i^2} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right] + c_i q_i + \tau_i(\rho_i - 1)^2 {r_i}^{\beta_i}\right]\right] \times \sum_{i=1}^{k} \left[\frac{1}{\alpha_i} \ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right]^{-1}$$

As α_i approaches zero in equation $TCU(q_i, \rho_i)$ reduces to $TCU(q_i, \rho_i) = \frac{TC(q_i)}{t_c} = \frac{A + \sum_{i=1}^{k}\left[a_i + \frac{h_i q_i^2}{2r_i\rho_i} + c_i q_i + \tau_i(\rho_i - 1)^2 {r_i}^{\beta_i}\right]}{\sum_{i=1}^{k} \frac{q_i}{r_i\rho_i}}$, where, $TC(q_i, \rho_i) = A + \sum_{i=1}^{k}\left[a_i + \frac{h_i q_i^2}{2r_i\rho_i} + c_i q_i + \tau_i(\rho_i - 1)^2 {r_i}^{\beta_i}\right]$. The average profit $\pi(q_i, \rho_i)$ per unit time is obtained by dividing t_c in $\pi_1(q_i, \rho_i)$. The total profit per cycle is $\pi_1(q_i, \rho_i)$.

$\pi_1(q_i, \rho_i) = Sales\ Revenue\ (SR) - Total\ Cost\ (TC) = \sum_{i=1}^{k}[(q_i - L_i) \times p_i] - \left[A + \sum_{i=1}^{k}\left[a_i + \frac{h_i q_i^2}{2r_i\rho_i} + c_i q_i + \tau_i(\rho_i - 1)^2 r_i^{\beta_i}\right]\right] = \sum_{i=1}^{k}[q_i \times p_i] - \left[A + \sum_{i=1}^{k}\left[A \times q_i^{(\gamma_i - 1)} + a_i + \frac{h_i q_i^2}{2r_i\rho_i} + c_i q_i + \tau_i(\rho_i - 1)^2 r_i^{\beta_i}\right]\right].$

Since L is zero. Hence the profit maximization problem is:

Maximize $\pi_1(q_i, \rho_i)$

$\forall\ q_i \geq 0, \rho_i \geq 0\ for\ i = 1,2,3 \ldots..k\ and\ all\ q_i s\ are\ integers$

Algorithm

Step 1: Set numerical values for the inventory parameters.

Step 2: Set $\frac{d\pi_1(q_i,\rho_i)}{dq_i} = 0\ and\ \frac{d\pi_1(q_i,\rho_i)}{d\rho_i} i = 1,2,3 \ldots \ldots k$ and solve by Lingo 13.0 for integer q_i and ρ_i. Find out the appropriate scenario and for that obtain multi-item profit per cycle.

Step 3: Check sufficiency condition graphically.

Optimization

The optimal integer ordering quantity q_i and publicity effort ρ_i per cycle can be determined by differentiating equation $\pi_1(q_i, \rho_i)$ with respect to q_i and ρ_i separately, setting these to zero. In order to show the uniqueness of the solution in, it is sufficient to show that the net profit function throughout the cycle is jointly concave in terms of integer ordering quantity q_i and publicity effort factor ρ_i. The second partial derivates of equation $\pi_1(q_i, \rho_i)$ with respect to q_i and ρ_i are strictly negative and the determinant of Hessian matrix is positive. Considering the following propositions:

Proposition 1: The net profit $\pi_1(q_i, \rho_i)$ per cycle is concave in q_i.

Conditions for optimal q_i

$$\frac{\partial\pi_1(q_i,\rho_i)}{\partial q_i} = p_i - \left(c_i + \frac{h_i q_i}{r_i\rho_i}\right) = 0$$

The second order partial derivative of the net profit per cycle with respect to q_i can be expressed as:

$$\frac{\partial^2\pi_1(q_i,\rho_i)}{\partial {q_i}^2} = -\left[\frac{h_i}{r_i\rho_i}\right] < 0,$$

Since $r_i\rho_i$> 0 and $h_i > 0$. So, the equation $\frac{\partial^2\pi_1(q_i,\rho_i)}{\partial {q_i}^2}$ is strictly negative.

Proposition 2: The net profit $\pi_1(q_i, \rho_i)$ per cycle is concave in ρ_i.

Conditions for optimal ρ_i

$$\frac{\partial\pi_1(q_i,\rho_i)}{\partial\rho_i} = \left(\frac{h_i {q_i}^2}{2r_i {\rho_i}^2}\right) - 2\tau_i(\rho_i - 1){r_i}^{\beta_i} = 0$$

The second order partial derivative of the net profit per cycle with respect to ρ_i can be expressed as:

$$\frac{\partial^2\pi_1(q_i,\rho_i)}{\partial {\rho_i}^2} = -\left[\frac{h_i {q_i}^2}{r_i {\rho_i}^3} + 2\tau_i {r_i}^{\beta_i}\right] < 0$$

Since$\left(\frac{h_i {q_i}^2}{r_i {\rho_i}^3}\right)$>0, τ_i > 0, r_i>0, it is found that $\frac{\partial^2\pi_1(q_i,\rho_i)}{\partial {\rho_i}^2}$ is strictly negative.

Proposition 1 and proposition 2 show that the second order partial derivatives of equation $\pi_1(q_i, \rho_i)$ with respect to q_i and ρ_i separately are strictly negative. The next step is to check that the determinant of the Hessian matrix is positive, i.e.

$$\frac{\partial^2\pi_1(q_i,\rho_i)}{\partial {q_i}^2} \times \frac{\partial^2\pi_1(q_i,\rho_i)}{\partial {q_i}^2} - \left(\frac{\partial^2\pi_1(q_i,\rho_i)}{\partial q_i \partial\rho_i}\right)^2 > 0$$

Hence, the Hessian matrix is $\frac{2h_i\tau_i r_i^{\beta_i-1}}{\rho_i} > 0$. Since, $\left(\frac{\partial^2\pi_1(q_i,\rho_i)}{\partial q_i^2}\right), \left(\frac{\partial^2\pi_1(q_i,\rho_i)}{\partial \rho_i^2}\right)$ shown in above propositions respectively and $\frac{\partial^2\pi_1(q_i,\rho_i)}{\partial q_i \partial \rho_i} = \frac{\partial^2\pi_1(q_i,\rho_i)}{\partial \rho_i\, \partial q_i} = \frac{h_i q_i}{r_i \rho_i^2}$

The objective is to determine the optimal values of integer q_i and ρ_i to maximize the net profit function. It is very difficult to derive the optimal values of integer q_i and ρ_i, to maximize the net profit function. But there are several methods to cope with constraints mixed integer non-linear optimization problem numerically. Lingo 13.0 software is used to derive the optimal values of the decision variables of the given mixed integer non-linear multi-item model.

Numerical Example

We consider ten different deteriorated items that need to be replenished jointly, namely items 1-10. The model is illustrated through the numerical example where the numerical data is given in Table 2.

Table 2: Numerical Data for the Example

$p_1 = Rs.125, p_2 = Rs.126, p_3 = Rs.127, p_4 = Rs.128, p_5 = Rs.129, p_6 = Rs.130, p_7 = Rs.131, p_8 = Rs.132, p_9 = Rs.133, p_{10} = Rs.134\ per\ unit$
$c_1 = Rs.100, c_2 = Rs.102, c_3 = Rs.104, c_4 = Rs.106, c_5 = Rs.108, c_6 = Rs.109, c_7 = Rs.110, c_8 = Rs.112, c_9 = Rs.115, c_{10} = Rs.116\ per\ unit$
$h_1 = Rs.5, h_2 = Rs.5.5, h_3 = Rs.6, h_4 = Rs.6.5, h_5 = Rs.7, h_6 = Rs.7.1, h_7 = Rs.7.2, h_8 = Rs.7.3, h_9 = Rs.7.4, h_{10} = Rs.7.5\ per\ unit\ per\ unit\ of\ time$
$r_1 = 1000, r_2 = 1050, r_3 = 1100, r_4 = 1150, r_5 = 1200, r_6 = 1210, r_7 = 1220, r_8 = 1225, r_9 = 1230, r_{10} = 1235\ units\ per\ unit\ of\ time$
$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = \alpha_9 = \alpha_{10} = 0$
$\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 2$
$a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = a_8 = a_9 = a_{10} = Rs.1\ per\ item$
$\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 = \tau_8 = \tau_9 = \tau_{10} = 2$
$A = Rs.200\ per\ order$

Fig. 2 represents two dimensional plotting of publicity effort factor and publicity effort cost PEC. Fig. 3 represents the three dimensional mesh plot of integer order quantity q_i, publicity effort factor ρ_i and net profit per cycle π_1. These figures show concavity of the net profit function per cycle. The optimal solution that maximizes $\pi_1(q_i,\rho_i)$, ρ_i^* and q_i^* are determined by using Lingo 13.0 version software and the results are tabulated in Table 3. Comparative analysis of a multi-item model with no publicity effort cost and the current multi-item model is shown in Table 3. It is observed that the multi-item net profit per cycle is 44.38% more than that of the multi-item net profit per cycle where no publicity policy is applied. So, considerable savings can be realized during the replenishment by the ordering of several different multi-items with implication of publicity policy. Comparative analysis of a single-product model and the present multi-item model is shown in Table 4. It is observed that the net profit is 46.36% more than that of the single-product model. So, multi-item retailers' publicity and integer ordering multi-item strategies are widely used in the real world for retailers' perspective.

Table 3: Optimal Values of the Proposed Model

Model	Iteration	t_c^*	ρ_i^*	q_i^*	PEC	π_1^*	π^*
MINLP Multi-Item	11 (2527)	33.02895	1.015625,1.012468,1.01002,1.008095,1.006565,1.006417,1.006277,1.005592,1.004449,1.004372, (10.07988)	5078,4639,4259,3924,3624,3602,3581,3375,3005,2977, (38064)	1821.329	410305.3	12422.59
NLP Multi-Item	493	33.06653	1.015615,1.012468,1.000019,1.008094,1.006563,1.006417,1.006276,1.005591,1.004450,1.004372 (10.06987)	5078.125,4638.942,4258.913,3923.811,3623.625,3601.837,3580.664,3374.93,3005.205,2976.96 (38063.01)	1821.092	410305.3	12408.47
% Change	-	0.1138	0.0993	0.0026	0.0130	0	0.1136
NLP Multi-Item	39	33.32431	-	5000,4581.818,4216.667,3892.308,3600,3578.873,3558.333,3356.164,2991.892,2964 (37740.06)	-	408484.2	12257.84
% Change	-	0.7796	-	0.8510	-	0.4438	1.3262

Table 4: Comparative Analysis of a Single-Product Model and Multi-item Model

Model	Programming	Item	Iteration	t_c^*	ρ_i^*	q_i^*	*PEC*	π_1^*	π^*
Single-Item	MINLP	1	2 (78)	4.999881	1.015624	5078	488.2346	62788.28	12557.96
	NLP		165	5	1.015625	5078.125	488.2813	62788.28	12174.24
	MINLP	2	2 (83)	4.363686	1.012468	4639	342.7605	55124.56	12632.56
	NLP		92	4.363634209	1.012468	4638.942	342.7438	55124.56	12323.50
	MINLP	\ 3	2 (119)	3.83341	1.010019	4259	242.9370	48534.58	12660.94
	NLP		62	3.833332838	1.010019	4258.913	242.9175	48534.58	12411.25
	MINLP	4	2 (85)	3.384614	1.0080994	3924	173.2992	42788.65	12642.1
	NLP		61	3.384613935	1.008094	3923.811	173.2663	42788.65	12439.91
	MINLP	5	2 (99)	3.000001	1.006564	3624	124.0819	37724.03	12574.67
	NLP		143	3.0000019	1.006562	3623.625	124.0132	37724.03	12411.25

	MINLP	6	2 (96)	2.957745	1.006417	3602	120.5835	37498.73	12678.15
	NLP		92	2.957745	1.006417	3601.837	120.562	37498.73	12516.99
	MINLP	7	2 (82)	2.916665	1.006277	3581	117.2798	37279.74	12781.63
	NLP		93	2.916666004	1.006276	3580.664	117.2363	37279.74	12622.69
	MINLP	8	2 (110)	2.739726	1.005592	3375	93.83397	33455.47	12177
	NLP		44	2.739726	1	3356.164	0	33361.64	12177.0
	MINLP	9	2 (88)	2.432433	1.004449	3005	59.89070	26786.93	10987.78
	NLP		50	2.432433	1	2991.892	0	26727.03	10987.78
	MINLP	10	2 (99)	2.4	1.004373	2977	58.32311	26534.32	11031.67
	NLP		50	2.4	1	2964	0	26476	11031.67
Total	MNLP/NLP	-	-	33.04/33.03	10.08/10.07	38064/38017.97	1821.22/3430.25	408515.3/408303.2	111711.8/121096.3
Multi-Item	MINLP	1-10	11 (2527)	33.02895	1.015625,1.012468,1.01002,1.008095,1.006565,1.006417,1.006277,1.005592,1.004449,1.004372,(10.07988)	5078,4639,4259,3924,3624,3602,3581,3375,3005,2977,(38064)	1821.329	410305.3	12422.59
% Change	-	-	-	0.0303/0.0303	0/0.0993	0/0.1209	0.006/88.3377	0.4636/0.488	8892.5837/874.8072
Multi-Item	NLP	1-10	493	33.32431	1.015615,1.012468,1.000019,1.008094,1.006563,1.006417,1.006276,1.005591,1.004450,1.004372 (10.06987)	5078.125,4638.942,4258.913,3923.811,3623.625,3601.837,3580.664,3374.93,3005.205,2976.96 (38063.01)	1821.092	410305.3	12408.4
% Change		-	-	0.8942	0.0992	0.0026	0.0131	0	0.1142

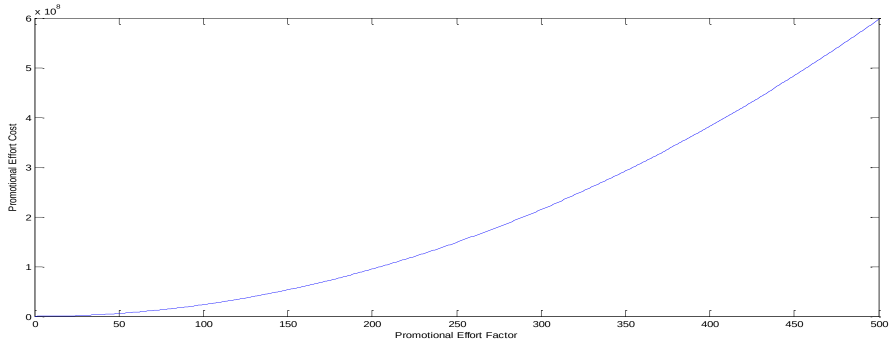
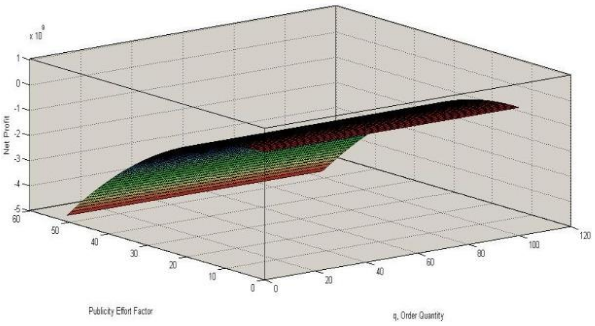
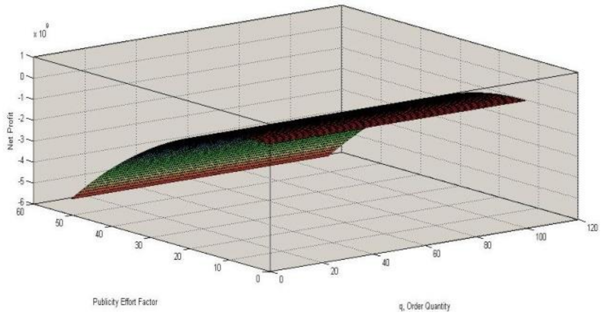


Figure 2: Two Dimensional Plotting of Publicity Effort Factor ρ_i and Publicity Effort Cost PEC



(a)



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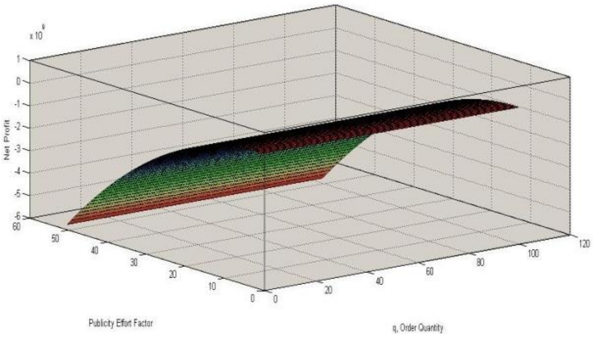

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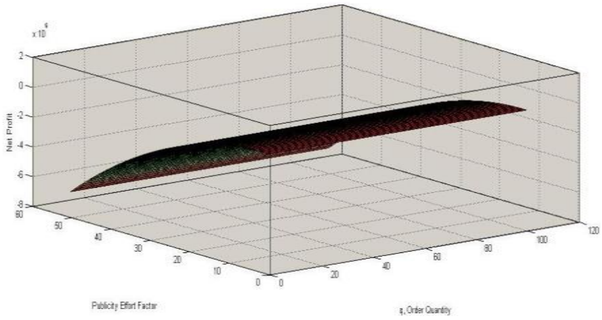
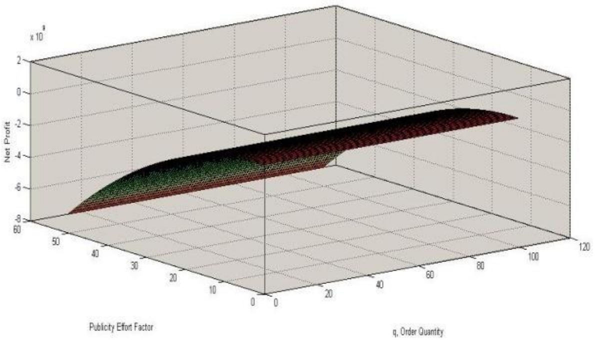
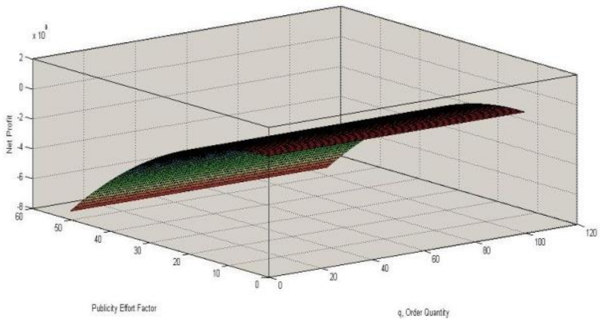


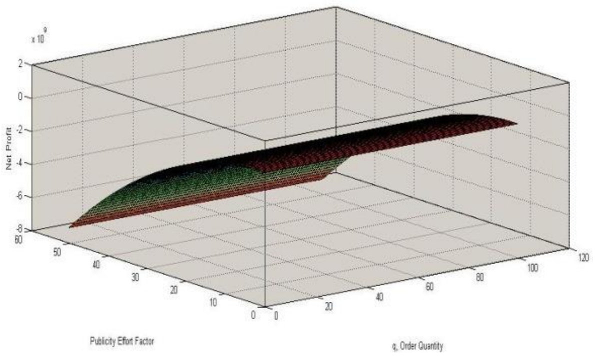
(d)



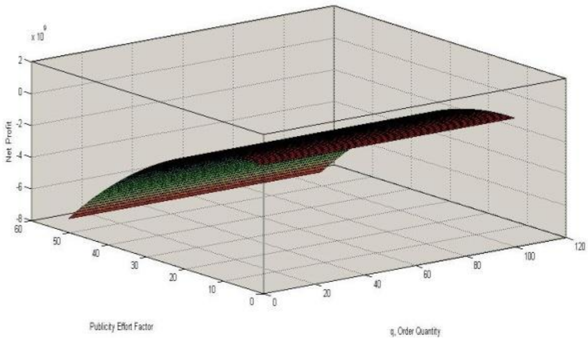
(e)



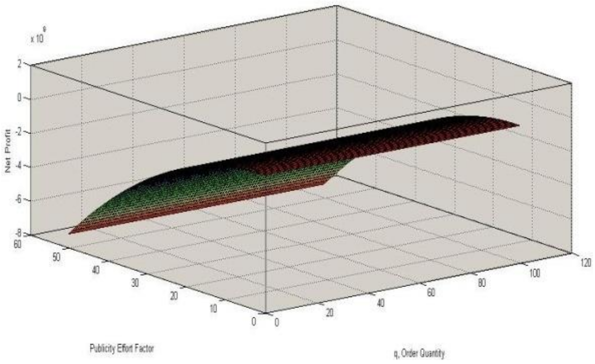
(f)



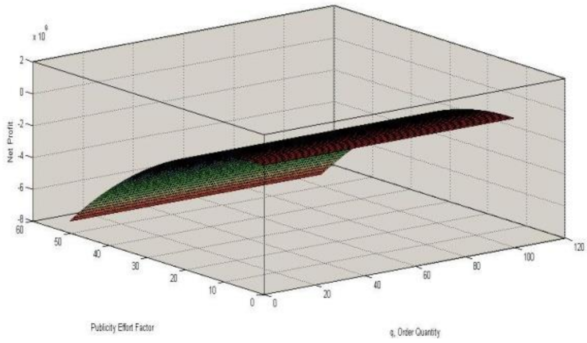
(g)



(h)



(i)



(j)

Figure 3: Three Dimensional Mesh Plotting of: (a) integer order quantity q_1, publicity effort Factor ρ_1 and net profit per cycle $\pi_1(q_1, \rho_1)$; (b) integer order quantity q_2, publicity effort factor ρ_2 and net profit per cycle $\pi_1(q_2, \rho_2)$; (c) integer order quantity q_3, publicity effort factor ρ_3 and net profit per cycle $\pi_1(q_3, \rho_3)$; (d) integer order quantity q_4, publicity effort factor ρ_4 and net profit per cycle $\pi_1(q_4, \rho_4)$; (e) integer order quantity q_5, publicity effort factor ρ_5 and net profit per cycle $\pi_1(q_5, \rho_5)$; (f) integer order quantity q_6, publicity effort factor ρ_6 and net profit per cycle $\pi_1(q_6, \rho_6)$; (g) integer order quantity q_7, publicity effort factor ρ_7 and net profit per cycle $\pi_1(q_7, \rho_7)$; (h) integer order quantity q_8, publicity effort factor ρ_8 and net profit per cycle $\pi_1(q_8, \rho_8)$; (i) integer order quantity q_9, publicity effort factor ρ_9 and net profit per cycle $\pi_1(q_9, \rho_9)$; (j) integer order quantity q_{10}, publicity effort factor ρ_{10} and net profit per cycle $\pi_1(q_{10}, \rho_{10})$

Sensitivity Analysis

It is interesting to investigate the influence of the major inventory parameters, $p_{i,} r_i, h_i, c_i, a_i, A, \tau_i \; and \; \beta_i$ on retailers' perspective mixed integer multi-item order quantity model. The computational results shown in Table 5 indicate the following managerial phenomena:

- $q_i, i = 1,2, \dots 10$ order quantities are highly sensitive, t_c the cycle length is highly sensitive, ρ_i the publicity effort factor are insensitive, PEC publicity effort cost per cycle is highly sensitive, π_1 the net profit per cycle and π the average profit per cycle are highly sensitive to the parameter p_i selling price for item i.
- $q_i, i = 1,2, \dots 10$ order quantities are moderately sensitive, t_c the cycle length is insensitive, ρ_i the publicity effort factor are insensitive, PEC publicity effort cost per cycle is highly sensitive, π_1 the net profit per cycle and π the average profit per cycle are sensitive to the parameter $r_i$the consumption rate for item i.
- $q_i, i = 1,2, \dots 10$ order quantities are sensitive, t_c the cycle length is sensitive, ρ_i the publicity effort factor are insensitive, PEC publicity effort cost per cycle is sensitive, π_1 the net profit per cycle is highly sensitive but π the average profit per cycle is moderately sensitive to the parameter $h_i$holding cost of item i per unit per unit of time.
- $q_i, i = 1,2, \dots 10$ order quantities are sensitive, t_c the cycle length is sensitive, ρ_i the publicity effort factor are insensitive, PEC publicity effort cost per cycle is sensitive, π_1 the net profit per cycle and π the average profit per cycle are sensitive to the parameter c_i purchasing cost for item i.

- $q_i, i = 1,2, \dots 10$ order quantities are insensitive, t_c the cycle length is insensitive, ρ_i the publicity effort factor are insensitive, PEC publicity effort cost per cycle is insensitive, π_1 the net profit per cycle moderately sensitive and π the average profit per cycle are less sensitive to the parameter a_i minor ordering cost of item i.
- $q_i, i = 1,2, \dots 10$ order quantities are insensitive, t_c the cycle length is insensitive, ρ_i the publicity effort factor are insensitive, PEC publicity effort cost per cycle is insensitive, π_1 the net profit per cycle and π the average profit per cycle are moderately sensitive to the parameter A major ordering cost per order.
- $q_i, i = 1,2, \dots 10$ order quantities are moderately sensitive, t_c the cycle length is insensitive, ρ_i the publicity effort factor are insensitive, PEC publicity effort cost per cycle is sensitive, π_1 the net profit per cycle is sensitive and π the average profit per cycle is moderately sensitive to the parameter τ_i of the publicity effort cost per cycle.
- $q_i, i = 1,2, \dots 10$ order quantities are highly sensitive, t_c the cycle length is insensitive, ρ_i the publicity effort factor are highly sensitive, PEC publicity effort cost per cycle is highly sensitive, π_1 the net profit per cycle and π the average profit per cycle are highly sensitive to the parameter β_i of the publicity effort cost per cycle.

Fig. 4 is about net profit per cycle variations with respect to inventory parameters. The profit increases highly with increase in per unit selling price and one parameter of publicity effort cost and then decreasing and the profit increases slightly with increase in consumption rate. The profit decreases with increase in holding cost per unit per unit time, purchasing cost per unit and minor ordering cost for item i, one parameter of publicity effort cost respectively but profit is remaining constant with increase in major ordering cost per order. Fig. 5 is about cycle length variations with respect to inventory parameters. The cycle length increases highly with increase in per unit selling price and it decreases slightly with increase in consumption rate. The cycle length decreases with increase in holding cost per unit per unit time, purchasing cost per unit for item i, it fluctuates with increase in the parameters of publicity effort cost but cycle length is remaining constant with increase in minor cost for item i and major ordering cost per order. This suggests that the retailer should work on the holding cost per unit per unit of time, purchasing cost per unit and minor ordering cost for item i. The retailer should put large order with implementing publicity strategy to save in ordering cost as a result profit of retailers can be increased significantly.

Table 5: Sensitivity Analyses of the Significant Parameters

Para-meter	Value	Iteration	t_c^*	$q_i{}^*$	$\rho_i{}^*$	PEC	π_1^*	π^*	% Char
p_i	136,137,138,139,140,141,142,143,144,145	2835	48.31552	7214,6653,6174,5758,5097,4704,4659,5390,5356,5324	1.030625,1.025022,1.020625,1.017124,1.014301,1.013983,1.013675,1.012580,1.010767,1.010580	7946.751	881889.1	18252.71	114.9349
	146,147,148,149,150,151,152,153,154,155	5183	63.63569	9455,8752,8159,7649,7204,7158,7113,6862,6441,6379	1.050625,1.041905,1.035019,1.029498,1.025015,1.024459,1.023921,1.022365,1.019831,1.019487	23373.89	1538890	24182.81	275.0597
	156,157,1	3162	78.9422	11831,10959,10	1.075625,1.0	54709.	238791	30248.8	481.98

	58,159,160,161,162,163,164,165			233,9616,9081,9020,8961,8683,8230,8149	63117,1.053201,1.045217,1.038705,1.03745,1.037013,1.034945,1.031641,1.031093	95	4	9	47
r_i	1100,1150,1200,1250,1300,1310,1320,1325,1330,1335	326	33.32991	5578,5075,4642,4642,3923,3897,3872,3648,3248,3216	1.014205,1.011383,1.009184,1.007446,1.006058,1.005927,1.005800,1.005169,1.004115,1.004045	1821.092	446330.1	13391.28	8.77995
	1200,1250,1300,1350,1400,1410,1420,1425,1430,1435	369	33.02815	6078,5511,5025,4600,4223,4193,4163,3922,3491,3456	1.013021,1.010473,1.008478,1.006895,1.005625,1.005506,1.005392,1.004807,1.003827,1.003763	1821.092	482354.9	14604.36	17.56
	1300,1350,1400,1450,1500,1510,1520,1525,1530,1535	195	33.02814	6578,5948,5408,4939,4523,4489,4455,4196,3734,3696	1.012019,1.009697,1.007872,1.006419,1.005250,1.005142,1.005037,1.004491,1.003577,1.003518	1821.092	518379.8	15695.09	26.34002
h_i	6,6.5,7,7.5,8,8.1,8.2,8.3,8.4,8.5	804	28.52674	4220,3917,3645,3396,3168,3154,3141,2966,2646,2625	1.013021,1.010549,1.008588,1.007014,1.005742,1.005624,1.005510,1.004918,1.003920,1.003858	1335.548	354000.0	12409.41	-13.7228
	7,7.5,8,8.5,9,9.1,9.2,9.3,9.4,9.5	576	25.11847	3611,3390,3186,2994,2814,2806,2798,2645,2363,2348	1.011161,1.009143,1.007514,1.006189,1.005104,1.005006,1.004911,1.004389,1.003503,1.003452	1022.199	311447.1	12399.13	-24.0938
	8,8.5,9,9.5,10,10.1,10.2,10.3,	1323	22.44109	3155.518,2988.623,2829.887,2677.906,2531.5	1.009766,1.008067,1.006679,1.005538,	809.2969	278111.0	12392.94	-32.218

	10.4,10.5			76,2527.19,2522.891,2388.067,2135.586,2123.755	1.004594,1.004511,1.004430,1.003963,1.004430,1.003963				5
c_i	101,103,105,107,109,111,113,116,117	1068	31.65353	4860,4441,4070,3742.784,3448.98,3428.288,3408.179,3204.445,2836.891,2810.251	1.01,1.01,1.00,1.00,1.00,1.00,1.00,1.00,1.00	1519.170	373143.9	11788.38	-9.05701
	102,104,106,108,110,112,114,117,118	493	30.10212	4660,4244,3882,3562,3274,3255,3235,3034,2668,2643	1.01,1.01,1.00,1.00,1.00,1.00,1.00,1.00,1.00	1256.793	337783.2	11221.24	-17.6752
	103,105,107,109,111,113,115,118,119	1193	28.58314	4453,4047,3694,338,3100,3082,3064,2864,2500,2477	1.01,1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00	1030.310	304219.5	10643.32	-25.8553
a_i	2,2,2,2,2,2,2,2,2,2	1949	33.18774	5078,4638,4258,3923,3623,3601,3580,3374,3005,2976	1.01,1.01,1.01,1.00,1.00,1.00,1.00,1.00,1.00	1821.092	410295.3	12362.86	-0.00244
	5,5,5,5,5,5,5,5,5,5	1408	33.18774	5078,4638,4258,3923,3623,3601,3580,3374,3005,2976	1.01,1.01,1.01,1.00,1.00,1.00,1.00,1.00,1.00	1821.092	410265.3	12362.95	-0.00975
	50,50,50,50,50,50,50,50,50,50	7394	33.18774	5078,4638,4258,3923,3623,3601,3580,3374,3005,2976	1.01,1.01,1.01,1.00,1.00,1.00,1.00,1.00,1.00	1821.092	409815.3	12348.39	-0.11942
A	300	1885	33.18774	5078,4638,4258,3923,3623,3601,3580,3374,3005,2976	1.01,1.01,1.01,1.00,1.00,1.00,1.00,1.00,1.00	1821.093	410205.3	12360.15	-0.02437
	500	413	33.18774	5078,4638,4258,3923,3623,3601,3580,3374,3005,2976	1.01,1.01,1.01,1.00,1.00,1.00,1.00,1.00,1.00	1821.092	410005.3	12354.12	-0.07312
	800	5209	33.18774	5078,4638,4258,3923,3623,3601,3580,3374,3005,2976	1.01,1.01,1.01,1.00,1.00,1.00,1.00,1.00,1.00	1821.092	409705.3	12345.08	-0.14623
	3,3,3,3,3,3,3,3,3,3	3448	33.17276	5052,4619,4244,3913,3615,3594,3573,3368,30	1.01,1.00,1.00,1.00,1.00,1.00,1.00,	1214.061	409698.3	12350.44	-0.14794

τ_i				00,2972	1.00,1.00				
	4,4,4,4,4,4,4,4,4,4	2515	33.17412	5039,4610,4237,3908,3611,3590,3569,3365,2998,2970	1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00	910.5458	409394.8	12340.79	-0.22191
	5,5,5,5,5,5.5,5,5,5,5	1155	33.14493	5031,4604,4233,3904,3609,3588,3567,3363,2997,2969	1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00	728.4366	409212.6	12346.16	-0.26631
β_i	1,1,1,1,1,1,1,1,1,1	249	33.18469	83125,64561,506875,40120,31950,31365,30801,26343,19366,18969	16.6,14.0,12.0,10.3,8.8,8.7,8.6,7.8,6.4,6.4	2013021	2421506	72970.58	490.1718
	3,3,3,3,3,3,3,3,3,3	16511	33.02842	5000,4581,4216,3892,3600,3578,3558,3356,2991,2964	1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00,1.00	1.657817	408485.9	12367.71	-0.44343
	3.5,3.5,3.5,3.5,3.5,3.5,3.5,3.5,3.5,3.5	5095	33.02817	5000,4581,4216,3892,3600,3578,3558,3356,2991,2964	1,1,1,1,1,1,1,1,1,1	0.5013591	408484.3	12367.75	-0.44382

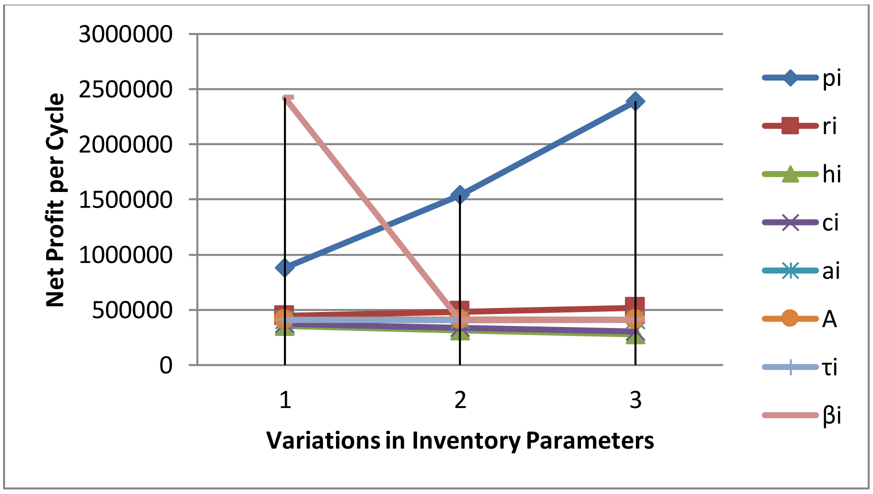


Figure 4: Changes in net profit per cycle with variations in inventory parameters

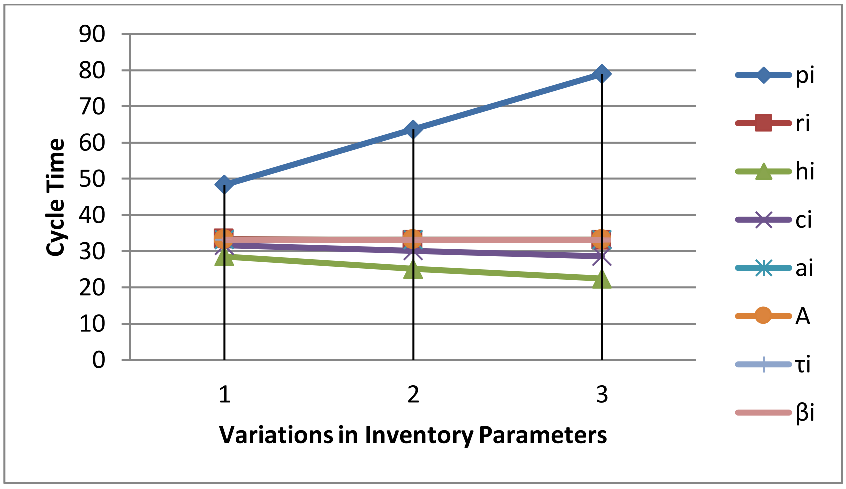


Figure 5: Changes in cycle time with variations in inventory parameters

Conclusion

In this model, a multi-item mixed integer EOQ model is introduced which investigates the optimal integer order quantity assumes that the inventory conditions govern the item stocked with publicity policy. This model provides a useful property for finding the optimal profit and ordering quantity where the deteriorated units are not lost. A mathematical EOQ model and algorithm is developed and satisfied the properties numerically. The economic order quantity and the net profit for the present model were found to be optimum respectively. Further, a numerical example is presented to illustrate the theoretical results, and some observations are obtained from sensitivity analyses with respect to the major inventory parameters to draw the managerial implications. The present model is a general framework in a multi-item retail-chain. The proposed model can be extended to incorporate preservation technology investment to control the deterioration of items in the retailer's inventory system.

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