

## ANALYSIS OF THE TECHNIQUE OF MINIMIZATION OF THE UNCERTAINTIES IN BUSINESS AND SENSITIVITY STUDY FOR THE OPTIMUM PERFORMANCE OF A FIRM

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### **Abstract**

*The studies related to the uncertainties in business have become the latest debate in the corporate world, and seeing its importance; it has been technically discussed and presented in this paper. The Theory and Modeling for the uncertainties in business have been given in the analytical form. The risk analysis and sensitivity analysis have been analytically discussed. The role of the probability model on the CDS to be looked after by the Administrator especially during uncertainty has been mathematically explained. The effects of uncertainties on business have also been discussed. The paper is expected to be useful to the new entrants in the field and also the senior business managers.*

**Keywords:** *Uncertainties in business, Measurement of uncertainty, Risk analysis, Strategic planning in uncertainty, Monte Carlo filtering.*

### **INTYRODUCTION**

The management performance and the financial position of the corporate firm are affected by the possible risk factors like – dependence on some of the overseas markets, major fluctuations in currency exchange rates, social and institutional changes in business areas, credit and accounts receivable recovery, and over concentration of production on a particular production site, and changes in consumer apparel spending and unseasonable weather. Also, factors like - Stagnant demand resulting from deteriorating economic conditions, Unforeseen changes in laws and regulations, Social turmoil due to terrorism, war, political upheaval, deteriorating civil order, and other causes, and Natural disasters including earthquakes contribute to the uncertainty factors. Uncertainty is a term used in various ways in many fields, like philosophy, statistics, economics, finance, insurance, psychology, sociology, and information science, the central idea in most of the situations being to make predictions of future events, which are mostly unknown, which in fact arise partially due to the changes in environments, and also due to ignorance e.g. Uncertainty refers to the lack of certainty, implying a state of having limited knowledge, leading to a situation, in which it is impossible to describe accurately the existing state, a future outcome, or even more than one possible outcome. The uncertainty is measured by assigning a set of possible states or outcomes so that the probabilities are assigned to each possible state or outcome, including the application of a probability density function to the continuous variable. In evaluating this, risk is also considered, which refers to the state of uncertainty, in which some of the possible outcomes have an undesired effect or a significant loss. In this way, risk is measured by assigning (i) a set of measured uncertainties, in which some possible outcomes are losses, and (ii) the magnitudes of those losses, including the loss functions over continuous variables.

For understanding the idea of the application of probability to risk, the example of choosing an MBA student from equal distributions of students of MBA and M. Tech., both from American and Indian universities, can be taken. The probability of the correct choice e.g. M.Tech, from Indian university =  $1/2 \times 1/2 = 1/4$  i.e. 25%, and so there is a risk of not getting the required one, which can be quantified =  $(1-1/4) = 3/4$  i.e. 75%. In this way, it can be increased to more parameters.

Sensitivity analysis and the Minimization Technique have been discussed in connection with the modeling, in order to optimize the performance.

### **Research Methodology**

The Analyst or the Manager takes the case under for study, and depending on the nature of the data, takes various steps: (i) Uncertainty and Risk Analysis, (ii) Statistical Tools and Mathematical Modeling, (iii) Sensitivity Analysis, and (iv) Minimization Technique.

### **UNCERTAINTY AND RISK ANALYSIS**

The subject of Risk versus Uncertainty has drawn the attention of various researchers. . However, the best understanding of this can be had from the explanation given by the great thinker, Mauboussin: in both risk and uncertainty, we do not know what is going to happen next, though in the first case, what the distribution looks like is known to us, whereas in the second case, the possible distribution is not known i.e. though the future is always unknown, it can not be classified as “uncertain”, just on the basis of that. In fact, this can be quantified in terms of the language of the statistical probability approach, which perfectly clarifies its falsity. In case of the tossing of the coin, we have the risk that it may not show the side of our interest like – head, but there is no uncertainty in its outcome that it will be either of the two outcomes – head or tail, So the probability of the happening of our choice is  $\left(\frac{1}{2}\right)$ . This is the risk or the uncertain future, but

not uncertainty, which occurs when we have no idea of the possible outcome, since the probability distribution is either unknown or so extremely large that functionally can be considered as unknown. This can be well understood by taking the example of the fiscal cliff, in which we know what the possible outcomes are, and also have a very good idea of their impact. Bartezzaghi and Verganti [1] have discussed that the topic of uncertainty has drawn the attention of various workers, who have studied the Managing of demand uncertainty through order overplanning. Bartezzaghi et al [2] have described a simulation framework for forecasting uncertain lumpy demand. Baker et al [3] and Bhagat, and Obreja[4] have also reported some recent very important studies on the topic.

### **STATITICAL TOOLS AND MATHEMATICAL MODELING FOR EVALUATING THE BUSINESS PERFORMANCE BASED ON THE SENSITIVITY ANALYSIS**

Chopra [5-8] have discussed in detail that the Statistical tools and Mathematical modeling have recently become very popular for understanding and discussing the various phenomena related to business management, as they are able to give quite an accurate idea about the quantization of the related parameters, and also the maximization and minimization of the required output. The Mathematical Modeling based on the Sensitivity Analysis has also been found quite useful in studying and quantifying the risk and uncertainty. Sensitivity analysis is the study of how the uncertainty in the output of a mathematical model or the system for studying the performance of the industry can be apportioned to the various sources of uncertainty in its inputs. The Uncertainty analysis, focuses more on uncertainty quantification and propagation of uncertainty. It can be easily visualized that ideally, uncertainty and sensitivity analysis should be carried in tandem. Helton et al [9], Saltelli and Annoni [10], and Bhagat et al [11] have discussed in detail

that the uncertainty can be measured analytically by Sensitivity Analysis or Uncertainty Analysis. This analysis is used for many purposes like – (i) Testing the robustness of the results of the system in the presence of uncertainty; (ii) Increased understanding of the relationships between input and output variables in a model; (iii) Uncertainty reduction by identifying the model inputs, which cause significant uncertainty in the output and should therefore be studied properly for enhancing the robustness; (iv) Searching for errors in the model based on quantifying the unexpected relationships between inputs and outputs; (v) Model simplification by fixing model inputs, which have no effect on the output; (vi) Enhancing communication from modelers to decision makers; and (vii) Finding regions of the input factors for maximizing or minimizing the model output based on the optimum criterion like the Monte Carlo filtering.

The locational source of the risk is also very important, which can be classified as: externally-driven or environmental risk, internally-driven or process risk, and decision-driven or information risk. In the analysis of the system, the risk has also to be located quite accurately. It can be of various types: (i) Strategic, in which the risk can be of the failure or success of the plans like - marketing strategy, consumer behaviour changes, or even political and regulatory changes; (ii) Financial, in which the risk can be of the failure or success of the financial control; (iii) Operational, in which there can be the risk of the human error or achievement design mistakes or sabotage; (iv) Commercial, in which the risk can be of the failure or success of the relationships of the key position holders and (v) Technical, in which there can be the risk of damage or enhancement of the assets due to the factors like equipment breakdown and fires. For an entrepreneur, making the annual budget of the company, many variables have to be considered like - tax rates, interest rates, inflation rates, headcount, operating expenses and other variables, which are having uncertainties of different degrees, It is impossible for even the wisest and most experienced Business manager to predict the deviations from expectations, quantitatively. Sensitivity analysis helps in studying the effect of the deviations of these variables from expectations, on the business, model, system, being analyzed, and identifying the variables causing the largest deviation. Also, the maximization or minimization of the function of many variables can be done by following the well known local method of taking the partial derivative of the output *Out* wrt input variable *Inp<sub>i</sub>*, as given below:

$$\left. \frac{\partial O u t}{\partial I n p_i} \right|_{I_0} \quad \text{---- (3.1),}$$

where  $I_0$  indicates that the derivative is taken at some fixed point in the space of the input, and

thus given the name – local. It has to be noted that this local method does not fully explore the input space, since they can evaluate only small perturbations, of single variable at a time. A more suitable method is based on the probabilistic approach, quantifying the input and output uncertainties as probability distributions, and dividing the output variance into two parts connected with (i) input variables, and (ii) combinations of variables. The working of the technique is based on measuring the amount of variance in the output resulting from that input, which can be studied in terms of the conditional expectations by considering a

model  $Out = f(Inp)$  for  $Inp = (Inp_1, Inp_2, \dots, Inp_i)$ , in which a measure of sensitivity of the  $i$ th variable  $Inp_i$  is given as:

$$Var_{Inp_i} \{ E_{Inp_{\setminus i}} (Out | Inp_i) \} \quad \dots (3.2),$$

where  $Var$  and  $E$  are respectively the variance and expected value operators, and  $Inp_{\setminus i}$  gives the set of all input variables except  $Inp_i$ . The Eqn. (2) measures the contribution  $Inp_i$

only to the uncertainty (variance) in  $Out$ , averaged over the variations in the other variables, and is termed as the first-order sensitivity index or the main effect index. In the same way, the total effect index is determined, which gives the total variance in  $Out$  resulting from  $Inp_i$  and its interactions with any of the other input variables. It is a common practice to

normalize and standardize both quantities by dividing them by  $Var(Out)$ . This approach considers the complete exploration of the input space, taking into account the interactions, and the nonlinear responses; and is based on the use of Monte Carlo methods. It has to be noted that the approach assumes that the input factors are independent of each other.

A central feature of the financial institutes is that the interest rate required to be paid by the firm on the loans taken during the periods of the firm's risk, as judged by the market, which occurs because of the factors affecting the value of the firm's assets, and also because of the uncertainty about the management of these assets, which in fact definitely affect the likelihood of default. Hence, a rational market has to incorporate managerial-generated uncertainty into its assessment of the firm's risk, for the accurate pricing of its securities. Holding constant a firm's fundamental risks, when there is more uncertainty about a management team's abilities or its future choices of actions, creditors should increase the interest rates they charge the firm. Uncertainty about management is likely to be highest when there is a new management team and should decrease over time as management's ability becomes known more precisely. Our empirical analysis suggests that CDS spreads on a firm's debt, loan spreads at origination, and the bond yield spreads at issuance are all significantly higher when the firm's CEO and/or the CFO are new in office than when they have been in office for three years. This pattern persists regardless of the reason for the management turnover. The sensitivity of the borrowing cost to CEO tenure also becomes more pronounced when the prior uncertainty about the manager is higher: if the CEO is not an heir-apparent, is an outsider, is younger, or has no prior relationship with the lenders.

The pricing of the CDS can also be done in terms of the probability model, in which, four inputs are taken. In case, no default events have taken place, the price of a CDS is equal to the sum of the discounted premium payments, and therefore, the CDS pricing models have to consider the possibility of a default taking place at some time between the effective date and maturity date of the CDS contract. It is usual to do this analysis by imagining the case of one year CDS with effective date  $t_0$  with four quarterly premium payments occurring at times  $t_1, t_2, t_3$ , and  $t_4$ . In case, the nominal for the CDS is  $N$  and the issue premium is  $c$ , then the size of the quarterly premium payments is  $Nc/4$ . For simplicity, it is assumed that the defaults occur on one of the payment dates, and so there can be following cases:

- (i) No default at all, so the four premium payments are made and the contract survives until the maturity date, or
- (ii) A default occurs on the first, second, third or fourth payment date.

The CDS can be priced by assigning probabilities to the five possible outcomes, and then calculating the present value of the payoff for each outcome, in which case, the present value of the CDS is the present value of the five payoffs multiplied by their probability of occurring.

If  $R$  is the recovery rate, or it survives without a default being triggered, in which case a

premium payment of  $(\frac{Nc}{4})$  is made. Clearly, the probability of surviving over the

interval  $t_{i-1}$  to  $t_i$  without a default payment is  $p_i$  and the probability of a default being triggered is  $(1 - p_i)$ . If the discount factors are respectively  $\delta_i$  to  $\delta_4$ ,

the probabilities  $p_1, p_2, p_3, p_4$  can be calculated by using the credit spread curve. Also, it can be taken that the probability of no default occurring over a time period from  $t$  to  $(t + \Delta t)$  decays exponentially with a time-constant determined by the credit spread, which is mathematically expressed as:

$$p = \exp\left\{-\frac{s(t)\Delta t}{(1 - R)}\right\} \quad \text{---- (3.3),}$$

where  $s(t)$  is the credit spread zero curve at time  $t$ . It has to be noted that the riskier the reference entity the greater the spread, and thus, more rapidly the survival probability decays with time.

The total present value ( $PV$ ) of the credit default swap is calculated by multiplying the probability of each outcome ( e.g.  $(1 - p_1), p_1(1 - p_2), \dots$ ) by its present value (e.g.

$N(1 - R)\delta_1, [N(1 - R)\delta_2 - \frac{Nc}{4}\delta_1], \dots$ ), and is given by taking their sum, and

in the following expression:

$$\begin{aligned}
 PV &= (1 - p_1)N(1 - R)\delta_1 \\
 &+ p_1(1 - p_2)[N(1 - R)\delta_2 - \frac{Nc}{4}\delta_1] \\
 &+ p_1p_2(1 - p_3)[N(1 - R)\delta_3 - \frac{Nc}{4}(\delta_1 + \delta_2)] \quad \text{--- (3.4)} \\
 &+ p_1p_2p_3(1 - p_4)[N(1 - R)\delta_4 - \frac{Nc}{4}(\delta_1 + \delta_2 + \delta_3)] \\
 &- p_1p_2p_3p_4(\delta_1 + \delta_2 + \delta_3 + \delta_4)\frac{Nc}{4}
 \end{aligned}$$

Calibration equations and propagation of uncertainty are very important for the uncertainty and risk analysis, and in practice, two classes of uncertainty propagation problems are commonly dealt with : (i) Interpolation in which the number of observations is exactly the same as the number of parameters in the calibration equation, which, however, is not quite suitable, because of the lack of the redundancy in the observations useful for ensuring that all the measurements are self-consistent without much probability of having committed any mistakes, and (ii) Alternatively, the method of the Least-squares fits, which is more appropriate, as in this case, many more observations than the minimum required are made, and the equation is finalized by using the least-squares fit. As the errors are likely to be random, there is a tendency of their canceling rather than adding in the Least-squares fits. In certain cases, the plots of the interpolated values of a parameter against the measured value of the parameter (K) are obtained to analyze the influence of an error in a fixed-point measurement. In practice, the difference between the two curves is called the fixed-point sensitivity coefficient

The accuracy of the technique of sensitivity analysis for minimizing the risk or uncertainty depends on the experience of the manager, who has to choose the analytic techniques which do not require high accuracy. It is now well accepted norm that the simple statistical models are generally found to be more reliable than more detailed and complicated models, dealing with highly complex situations in the market, and in fact are really preferable for the marketers and the business managers, who analyze the awareness and preference data more seriously and accurately. In such cases, the predictive analysis (5) is also very useful for case under study. To get the optimized results, the business manager should try to make multiple guesses and not just aim at the making of a single right guess, considered to happen most likely; since the placing of many small bets on many options, triggers the innovative process (7), which is also similar to prediction. In addition, it is advisable to look for and depend on the predictable elements of the situation, by concentrating to find out and locate the few participants with really good influence, who can assist the business manager to take the appropriate decision and action. Besides, the manager has to focus on his evaluation of the initiatives on the inputs, along with the outputs. It is evident that he has to counter the adverse effect of the randomness by the quality of the process for its planning and execution, and not by just relying on the actual outcome of the project, whether favourable or not. Also, he must remain very agile in order to be able to respond quickly, so that he can fix his reputation for being highly trustworthy person.

### 3.1 Sensitivity Analysis

Sensitivity analysis of our mathematical model or system can be done by following two well known approaches –

(i) Gaining clear knowledge about the various parameters that are important to the model or the system, by identifying the important connections between observations, model inputs, and predictions or forecasts (12,13).

This in fact implies understanding the observations i.e. measurements of dependent variables, which are most and least important to model inputs in the form of the parameters representing system characteristics or excitation. This obviously, can be done by noting as to which model inputs are most and least important to predictions or forecasts, and what observations are most and least important to the predictions and forecasts. However, it is interesting to note that these results are really surprising leading to facing problems.

(ii) Gaining complete information about the various parameters, which are important to calculated measures of uncertainty i.e. studying of how the uncertainty in the output of a mathematical model or system in terms of numbers or other parameters, can be connected to different sources of uncertainty in its inputs (14). Sometimes, uncertainty analysis is applied, which focuses on uncertainty quantification and propagation of uncertainty. To get the optimum results, sensitivity analysis and uncertainty analysis should be applied together.

Therefore, the sensitivity analysis is just a technique used to find quantitatively, as to how different values of an independent variable influence a particular dependent variable under certain assumptions. It has to be noted that this technique is used within specific boundaries, depending on one or more input variables, such as the effect of changes in interest rates on bond prices and Real Estate.

Sensitivity analysis, also called as simulation analysis, is used to predict the outcome of a decision in a certain range of set of variables. The analyst can determine how changes in one variable affect the outcome by creating a given set of variables.

#### **Application of Sensitivity Analysis for case study**

The sensitivity analysis can be applied by the manager or analyst by taking the case under study. If we consider the case of 1%, 1.5% or 1.75% increase in interest rates on the sale of bonds and their prices, and find that transactions decrease by 5, 25 or 50 for each set of 100 transactions, we can conclude that the sales are highly sensitive to changes in interest rates. Also, if the prices are affected only by half an amount, we can say that the bond prices are only half as sensitive as the bond sales to the change of interest rate.

There are two other methods - Local methods and Variance-based methods

Local methods are based on taking the partial derivative of the output Y with respect to an input factor  $X_i$ . Eqn. (3.1) can be more appropriately based on this technique

$$\left| \frac{\partial O u t}{\partial I n p i} \right|_{I_0}$$

where  $I_0$  indicates that the derivative is taken at some fixed point in the space of the input, and

thus given the name – local. It should be noted that this local method does not fully explore the input space, and is limited to examining small perturbations for one variable at a time.

The Variance-based methods are based on the probabilistic approaches, quantifying the input and output uncertainties as probability distributions, and decomposing the output variance into parts corresponding to the input variables and in some cases their combinations.

### **Minimization Technique**

Minimization is the Technique of Adaptive Stratified Sampling, which is used in clinical trials, and in fact aims to minimize the imbalance between the numbers of variables (e.g. Employees or groups in a firm) of output in each corresponding group over a number of factors. Sometimes, blocked randomization technique is applied to stratify variables according to a number of factors, and also using a separate randomization list for each variable. The advantage in this case is that each randomization list is created in such a way that after every block of a fixed number of variables, there is an equal number in each group in the case under study like factory or firm. However, the limitation of this technique is that the number of lists increases exponentially with the number of stratification factors. However, this problem is overcome by computing the imbalance within each factor in case the variable is allocated to a particular group. Finally, the overall imbalance of the study is calculated by taking the summation of all the imbalances. The group, which is able to minimize the imbalance, is chosen directly and thus the minimization technique is really useful in achieving the optimum result. If the factors are of varied importance, then it is better to give various weights to different groups so as to give some factors more importance than others. It is now well established that the minimization technique is able to maintain a better balance than the traditional blocked randomization, and its advantage rapidly increases with the number of stratification factors.

### **UTILITY OF PLANNING IN UNCERTAINTY PERIOD OF THE FIRM**

During the periods of uncertainty, because of the uncertain political conditions, mass unrest, or tension on the border etc, the business managers have to be extra vigilant and cool while planning, especially in the global economy, in which periods of consistent economic growth have led to the rising unemployment, increased costs, reduced disposable income and most importantly, enhanced number of risks. The managers have to be vigilant in the sense that they have to keep themselves updated with the latest economic indicators and forecasts by reading and understanding the minds and decisions of the economists, and the government secretaries. This should be followed by revisiting the earlier business plans, and the latest stocks and cash flows, and reviewing the current projects and plans, so as to take the corrective actions for each minor effective step. The communication of the managers with the firm employees is also of paramount importance, which can be done by the Management ensuring the employees to be fully aware of issues with implications for them, required for taking any legitimate change initiatives for reducing the costs and thereby improving the financial position of the company. Some effective steps like the consideration of outsourcing some activities, and assessing exposure to the known risks and dependencies should also be taken. In addition, great attention should be paid to the factors like - changing of the Scenarios, need to innovate, cost cutting, and keeping a constant watch on the cash position, and all this should be done by taking the next line of the managers in deep consultations in the regularly arranged meetings and seminars, where outside experts may also be invited for the expert advice and opinion.

### **CONCLUSION**

It is important to note that before taking any concrete steps in case of uncertainty, we must try to ascertain the level of uncertainty e.g. Level 1, in which there is a Clear-Enough Future; Level 2, in which there are Alternate Futures; Level 3, in which there is a Range of Futures; and Level 4, in which there is True Ambiguity.. In the first case, the managers are able to develop a single forecast of the future which can be taken as the accurate basis for strategy development, and the uncertainty is irrelevant to taking the strategic decisions. In the second case, the future is counted as one of a few alternate outcomes, and the manager is not able to identify which outcome will occur. In case of the level 3 uncertainty, the manager can identify a range of potential futures, and



the actual outcome may lie anywhere along a continuum bounded by that range. The level 4 refers to the case of the multiple dimensions of uncertainty interacting and creating an environment, which is totally unpredictable. Unlike in level 3 situations, the range of potential outcomes and the scenarios within that range cannot be identified. It might not even be possible to identify, much less predict, all the relevant variables that will define the future.

Sensitivity analysis of our mathematical model or system has been done by following two well known approaches. Application of Sensitivity Analysis for case study has also been done by taking an example. Minimization Technique has been briefly discussed, as the Technique of Adaptive Stratified Sampling, which is used in clinical trials, and in fact aims to minimize the imbalance between the numbers of variables.

It is worthwhile to note that the strategic planning (8), useful in normal situations is not so effective in case of uncertainty. In fact, it can be dangerous, as it can give very erroneous results leading to the taking of wrong corrective measures for reaching the correct decisions. This is primarily due to the fact that this approach leads executives to judge uncertainty in a binary way by just assuming that the world is either certain, and hence precise predictions can be made about the future, or uncertain, which is a totally unpredictable case. The processes of Planning and Capital-budgeting, which need point forecasts, enforce the business managers to ignore the underlying uncertainties in their cash flows, and thus force them to underestimate uncertainty so that a compelling case for their strategy can be made. This in turn, leads to those strategies which can not defend against the threats, and also are not able to take advantage of the opportunities likely to be provided by those higher levels of uncertainty. It is well known that even before the dawn of the computers and the related software, the industry and corporate experts were able to arrive at astonishingly accurate decisions within the range of possibilities. On the contrary, in case we consider that the world is entirely unpredictable, managers tend to completely abandon their analytical and reasoning skills required for their traditional planning processes, and take important strategic decisions just on the basis of their gut instinct. The managers who are afraid of taking risks in the cases of uncertainty, and so do not take decisions by trusting their gut instinct, just suffer from decision paralysis, which is disastrous for the business, because of the fact that they refrain from taking critical strategic decisions concerning the products, markets, and technologies to be developed by their company, and instead focus on other parameters like quality management, and cost-cutting programs, which are only partially productive in such situations. Thus, we can conclude that the efficient managers, in these cases, have to tailor their strategies to bring relief and benefit to their company. Thus, we can conclude that the good managers have to use a variety of tools in tandem like – important steps of strategies, risk analysis, sensitivity analysis, extrapolation technique, least square fits, and above all gut instincts, for achieving good results in the environment of uncertainty.

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