THE EFFECT OF INCREASING THE NUMBER OF INDICATORS ON THE PERFORMANCE OF DEA

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Abstract:

In this paper it is assumed n observation decisions units with m inputs and s outputs are exist. Sensitivity analysis is for the case when the index added to the collection. Adding a new index to indexes and we find the range for this index in a way that performance of efficient unit and performance of inefficient units to be maintained. To create this area and avoid resolve problem use sensitivities of linear programming techniques.

Keywords: Data envelopment analysis, efficiency, input and output, sensitivity analysis

1. Introduction

For a set of n decision unit n with m inputs and s outputs, performance of all decision units has been determined. Now, we want to change the number of units to examine performance Indexes. Here the question arises whether it is necessary the performance model solved with this change again? For example by adding a co index in the cover form of the question, aspect of issue will increase that resolve the problem in higher aspect it's not economical. Therefore, research should be done that can help lower computational process, identified performance changes. That the studies are doing as sensitivity analysis. Sensitivity analysis has been always considered by researchers. In this context, Churns and Cooper 1968 discussed data changes and then in 1985, due to data changes in both the DEA linear programming problem constraints expressed the need for new algorithms [3].They develop in 1992 the technique of sensitivity analysis in the case of simultaneous changes in all inputs and outputs for a specific DMU [6]. In addition, in 1996 investigated the sensitivity and stability analysis of performance classification that before 1992 were involved in this issue in particular [5].Due to changes in the input and output a stable area was define that Zhou and Seaford studied unique variations of each input or output and the corresponding changes in the DMU to calculate the stable area and changes to the form of deteriorative changes for DMU under evaluation and ameliorative changes in other DMU considered. So, the efficiency of DMU under evaluation maintained [10].Churns and et al 1992 with help of L1 and $L\ \infty$ provide models for calculating the stability radius [5].Sensitivity and stability analysis in DEA models inverse classification performance was evaluated by Jahanshahloo et al 2005 [8].Another issue that can be examined in the sensitivity analysis in the case of adding or reducing the number Indexes that in present paper, one method is suggested. In part 2 of this article, we describe some definitions and needed models. And in Section 3 with the addition of an index of input (output) study its effect on the value of performance units and then a range for Added index find such that DMU be inefficient[7]. In section 4 for clarity presented some examples.

2. Concepts

N, DMU with m inputs and s outputs will be assume, which X is an m × n input matrix and Y is an s × n output matrix with positive elements. Collection} 0 ≥ y that is generated by 0 ≥ x.{(x, y),is called the production possibility set (PPS).One observation (DMU) says effectively if and only if not defeated by any point of PPS and otherwise DMU is inefficient.CCR model based on efficient technology with fix scale (CRS) presented by Chuns, Cooper and Rods After that Banker, Churns and Cooper designs a model base on efficient technology with variable scale (VRS) that became known as the BCC model. Possibility of producing these two models, are shown as follows:

$T= \{(x, y) | x \lambda \le x, y\lambda \ge y, \lambda \in \Lambda\}$

$\Lambda ccr = \{\lambda | \lambda \ge 0\}$, $\Lambda BCC= \{\lambda | \lambda=1, \lambda \ge 0\}$

Coverage form of CCR model of the input nature is as follows:

$\text{Min}\theta$

s.t.

$X\lambda \le \theta X p$

$Y\lambda \ge Y p$

When optimal solution of a linear programming problem is obtained, changes in problem are usually one of the following and examined by sensitivity analysis [2]

Changes in the cost vector:

If the change in the objective function coefficient of a non-basic variable occurs, in the optimal schedule, only the variable coefficients in the objective function row will change. But if the change in one of the objective functions coefficients of the basic variables applied, in the optimal schedule, in addition coefficients of all other variables in the simplex table will also affect the optimal values[9].

Change in Vector Resources:

By changing the right of the simplex table possibility, if issue will question but hasn't affected on optimality

Changes in technology matrix:

Changes in the coefficients of a non-basic variable, affect on the variable in the objective function row optimal table. But if change in the coefficients of the basic variables, basic Matrix and Then the optimal schedule would change.

Add a new activity:

By information data, Z j-C calculates added corresponding variable, if the value was positive (in the case of Min)new variable enter the basic and must continue to achieve optimality simplex method. Otherwise, the current answer is optimal, and this variable by a zero value appear in optimal response.

Add a new limit:

If optimal respond of main issue, satisfy added limits, this point is optimal respond of new solution, and otherwise, use the dual simplex method to find the new optimal solution.

Increase the number of performance indicators:

With increasing index,(increasing the number of inputs and outputs)probability of being efficient be more. So by increasing the number of index, in coverage form, number of model increase and optimal respond of model can't be better, so performance goes up. Now, we evaluated the impact of increasing the efficiency of a DMU.

1-3 impact of adding input parameters on the performance

As it was said by adding an index to a coverage form a constraint is added.

$\theta^{*}=\text{Min } \theta$

$\text{S t.}\Sigma\lambda_j x_{ij} \le \theta x_{ip,\ j=}$ 1, 2...m

$\Sigma\lambda_j x_{rj} \geq yrp$ j=1, 2… s
$\Sigma\lambda j\ x_{m+1,\,j} \leq \theta x_{m+1,\,p}$
$\lambda \in \Lambda$
$\Lambda_{CCR=}\ \{\lambda\lambda \geq 0\}, \Lambda_{BCC=}\ \{\lambda\ 1\lambda = 1 \& \lambda \geq 0\}$

Suppose S be the possible region model for CCR (BCC) covering form before adding new constraints and S' be a possible region after adding reservation. By adding constraints, one of the two following conditions occurs:

- Φ = S ', in this case due to the intractable nature of issue, adding new constraints it is not possible. This does not happen in the CCR model, because this model with adding constraints as a new index, remain possible.

S '≠ φ, in which case we have the following cases:

Theorem 1 Add an index entry does not affect the performance of efficient DMU.

By argument we add an index S '⊆ S So θ * θ `* ≮.Therefore, θ * ≥ θ `* because DMU is under evaluation (DMUP) before adding the efficient index, According to the above equation and the fact that the optimal model to cover the interval [0, 1) is true, we can conclude that = 1θ `*.Dumb performance is preserved so the addition the index. If B is a constraint the basic optimization form the base coat before adding issue new resource vector (after adding constraint) is defined as follows:

$$B_{new} = \begin{bmatrix} B & 0 \\ U & -1 \end{bmatrix} \quad , \quad b_{new} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

U = [- x $_{m+1,j1}$ …– x $_{m+1,\,jk}$ …. 0 ….0 x $_{m+1\,,\,p}$]

K number of variables λ correspond to the basic and auxiliary variables corresponding to issue zeros in matrix U is the basic. B new and B, respectively, of order m + s and m + s +1 and U is matrix of order (m + s). In this case 1 - (Bnew) is calculated from the following equation:

B^{-1} 0
$(B_{new})^{-1} = UB\ [^{-1}\quad -1]$

And on the other hand:

$$(Z_J - C_{J})_{new} = (z_j)_{new} - c_j = \begin{bmatrix} C_B & 0 \end{bmatrix} \begin{bmatrix} B^{-1} & 0 \\ UB^{-1} & -1 \end{bmatrix} \begin{bmatrix} a_j \\ -x_{m+1+j} \end{bmatrix} - C_J = Z_J - C_J$$

Similarly, this equation can be writing for other variables. So according to these relations it can be seen that the optimal schedule by adding new limits and efficient units and ineffective unit has no effect on optimality. Therefore, to maintain the efficiency of inefficient units is sufficient to do condition for the feasibility of the optimal schedule. To investigate the feasibility of adding a new constraint, we do the following:

$$(B_{new})^{-1} = \begin{bmatrix} \bar{b} \\ U\bar{b} \end{bmatrix}$$

Because optimal respond is primary issue is possible so $\bar{b} \geq 0$

According to $U\bar{b}$ value following cases expressed:

Theorem 2 if the vectors xm +1 DMUP inefficient to maintain performance levels, then the following applies:

$$X_{m+1,p} \geq \frac{\sum_{j=1}^{n} \lambda_j^* x_{m+1,j}}{\theta^*}$$

According to retain the value argument DMUP performance and optimal schedule remains constant, to remain possible optimal respond after adding the new constraints we have

$U\bar{b} \geq 0$

According to assumption DMUP is inefficient so (-XP, YP)is not included in the base. Without reduce the generalization problem 1.... λ2 ..., λKλ considered as part of the initial base and U-matrix can be defined as follows:

$U = [\,-X_{m+1,j}\ \ 0 \ldots 0\ X_{m+1,p}\,]$, t = 1,2,…..,k , j ≠p

And we have:

$$\bar{b} = \begin{bmatrix} \lambda_j^* \\ S_t^* \\ {}^*\theta \end{bmatrix}$$, j = 1, 2… k وj≠p و t = 1, 2,…, m+s – (k+1)

So $U\bar{b} = -\sum_{j=1}^{k} \lambda_j^* x_{m+1,j} + \theta^* x_{m+1,p}$ ≥0therefore $X_{m+1,p} \geq \frac{\sum_{j=1}^{n} \lambda_j^* x_{m+1,j}}{\theta^*}$,due to none basic we have$_j$ λ (j=k+1....n):

$$X_{m+1,p} \geq \frac{\sum_{j=1}^{n} \lambda_j^* x_{m+1,j}}{\theta^*}$$

With the assumption there is no issue (2) to maintain the efficiency of an inefficient DMU we have theorem3:

Theorem 3: Assume that the sentence does not establish theorem 3.Then the performance of an inefficient DMU if adding constant input parameters, then the following relations is established.

1 $U\bar{b} < 0$

2 $a_{m+1,L} - UT_L > 0$

3 $\bar{b} + \frac{U\bar{b}}{a_{m+1,L} - UT_L} \Gamma_L \geq 0$

That $\Gamma_L = B^{-1} a_l$ and variable L incoming to base

Where the argument of the case (2) does not establish so:

$$X_{m+1,p} < \frac{\sum_{j=1}^{n} \lambda_j^* x_{m+1,j}}{\theta^*}$$

There for:

$U\bar{b} < 0$ (5)

For that, the optimal value of the objective function does not change should

$\exists j \in NB$ $Z_J - C_J = 0$

Suppose ZJ - CJ = 0 are L non basic incoming variable to base:

So the base is defined as follows:

$$B' = \begin{bmatrix} B & a_L \\ U & a_{m+1,L} \end{bmatrix}$$

Inverse matrix is calculated from the following equation:

$$(B')^{-1} = \begin{bmatrix} B^{-1} + \Gamma(a_{m+1,L} - U\Gamma_L)^{-1}UB^{-1} & -\Gamma_L\left(a_{m+1,L} - U\Gamma_L\right)^{-1} \\ -(a_{M+1,L} - U\Gamma_L)^{-1}UB^{-1} & (a_{m+1,L} - U\Gamma_L)^{-1} \end{bmatrix}$$

Due to the no effects of adding constraints on optimality issues, we have applied possibility condition:

$$(B')^{-1}b_{new} = \begin{bmatrix} \bar{b} + \dfrac{(U\bar{b} - y_{s+1,P})\Gamma_L}{a_{m+1,L} - U\Gamma_L} \\ \dfrac{-U\bar{b}}{a_{m+1,L} - U\Gamma_L} \end{bmatrix}$$

So

$$\bar{b} + \frac{(U\bar{b} - y_{s+1,P})\Gamma_L}{a_{m+1,L} - U\Gamma_L} \; \Gamma_L \geq 0$$

$$\frac{-U\bar{b}}{a_{m+1,L} - U\Gamma_L} > 0$$

Because <0, then we have in the above equation:

$$(7) \qquad a_{m+1,L} - U\Gamma_L > 0$$

Circumstances of the case, for all other variables in the rows of the table, the optimal value are zero apply and community obtained ranges that is desirable area. In general of conditions community of Theorems 2 and 3 to come, if there is index value of (m +1) in it, Maintain the efficiency of a given inefficient DMU.

3-2 The effect of adding an output index on function

By adding an output index issue with covering form, is converted as following:

$$Min\ \theta$$

$$s.t \qquad -\sum_{j=1}^{n} \lambda_j x_{ij} - s_i^{-} + \theta x_{ip} = 0, \quad i=1, 2\ldots, m$$

$$\sum_{je1}^{n} \lambda_j y_{rj} - s_r^{+} = y_{rp}, \qquad r=1, 2,\ldots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{s+1,j} - s_{s+1}^{+} = y_{s+1,p} \qquad (8)$$

$\lambda \in \Lambda$, $s^- \geq 0$, $s^+ \geq 0$.

$$A_{CCR} = \{\lambda : \lambda \geq 0\}, \quad A_{BCC} = \{\lambda : 1\ \lambda = 1 \ \& \ \lambda \geq 0\}$$

Suppose S '≠ ∅. With regard to the first issue the optimal basis B (before adding issue new constraint) we have:

$B_{new} = \begin{bmatrix} B & 0 \\ U & -1 \end{bmatrix}$, $b_{new} = \begin{bmatrix} b \\ y_{s+1,P} \end{bmatrix}$

$U = [y_{s+1,j1} \dots y_{s+1,jk} \dots 0..0]$

K number of correspond variables to basic λ and existing zeros in the matrix U corresponding basic and auxiliary variables and θ is the basic issue. B new and B, respectively is, of order m + s and m + s +1 and U matrix 1. (M + s).Theorem4 does not affect on adding an output index of an efficient DMU. The argument proof is similar theorem1.

Theorem 5: If the vector YS +1 maintain performance of inefficient DMUP Always applies to the following equation:

$0< y_{s+1,P} \le U\bar{b} - y_{s+1,P} \sum_{j=1}^{n} \lambda_j^* y_{a+1,j}$

Similar arguments to prove theorem 2:

$(B^{-1}b)_{new} = \begin{bmatrix} \bar{b} \\ U\bar{b} - y_{s+1,P} \end{bmatrix}$

$$U\bar{b} - y_{s+1,P} = \sum_{K=1}^{L} \lambda_K^* y_{s+1,K} - y_{s+1,P} \ge 0$$

So $\sum_{j=1}^{n} \lambda_K^* y_{s+1,K} \le y_{s+1,P}$ due to non-basic j (j=k+1… n)λ وشرط و Y≥0, Y≠0 we have

$0< y_{s+1,P} \le \sum_{j=1}^{n} \lambda_j^* y_{a+1,k}$

If Theorem5 wasn't true, In order to maintain the efficiency of a DMU is inefficient given theorem 6 expressed:

Theorem 6 If the sentence theorem 5 does not establish then if add an index of the inefficient performance DMUP remains constant, then following relations is established.

1. $U\bar{b} - y_{s+1,P} < 0$
2. $a_{m+1,L} - U\Gamma_L > 0$
3. $\bar{b} + \dfrac{(U\bar{b} - y_{s+1,P})\Gamma_L}{a_{m+1,L} - U\Gamma_L} \quad \Gamma_L \ge 0$

That $\Gamma_L = B^{-1} a_l$ and variable L is incoming to base. We have similar argument to theorem 3.

$U\bar{b} - y_{s+1,P} < 0$

(9)

$$(B')^{-1} b_{new} = \begin{bmatrix} \bar{b} + \dfrac{(U\bar{b} - y_{s+1,P})\Gamma_L}{a_{m+1,L} - U\Gamma_L} \\ \dfrac{-U\bar{b}}{a_{m+1,L} - U\Gamma_L} \end{bmatrix}$$

$$\bar{b} + \frac{(U\bar{b} - y_{s+1,P})\Gamma_L}{a_{m+1,L} - U\Gamma_L} \quad \Gamma_L \ge 0 \qquad (10)$$

$$\frac{-U\bar{b}}{a_{m+1,L}-U\Gamma_L} > 0$$

According to equation (3-7) and the above equation is obtained:

$a_{m+1,L} - U\Gamma_L > 0$

According to equation (3-7) and the above equation is obtained:

And from community of obtain area, wanted area determine to add output index. The amount of output parameters (m +1) the value of the efficiency of a DMU to preserve inefficient public provision of theorems 5 and 6 is obtained.

-3 To add an index in order to being efficient of an inefficient DMU

Value index (m +1) determined such that inefficient DMUP, become ineffective and the classification of performance of other DMU does not change.

Theorem 7: The vector Xm +1, which leads to inefficient the inefficient DMUP the following relationship, applies:

1 $U\bar{b} < 0$

2 $-x_{m+1,P} - U\Gamma_P > 0$

3 $\left|\frac{Z_P - C_P}{-(-x_{m+1,P} - U\Gamma_P)}\right| < \left|\frac{Z_j - C_j}{-(a_{m+1,P} - U\Gamma_j)}\right| \quad \forall\, j \in NB$

Added index argument must be such that Added index argument must be such that make inefficient Pλ, efficient, therefore, the optimal amount will change, and the necessity of this change is that the previous optimal solution in the new provision does not apply, it means:

$U\bar{b} < 0$

We assume Pλ in the first iteration after adding a new provision enters the base, so:

$$\frac{Z_P - C_P}{-(-x_{m+1,P} - U\Gamma_P)} = \text{Min}\left\{\frac{Z_j - C_j}{-(a_{m+1,P} - U\Gamma_j)} : -(a_{m+1,P} - U\Gamma_j) < 0,\ \forall j \in NB\right.$$

From above equation is obtained by the following two conditions:

$-x_{m+1,P} - U\Gamma_P > 0$

$$\left|\frac{Z_P - C_P}{-(-x_{m+1,P} - U\Gamma_P)}\right| < \left|\frac{Z_j - C_j}{-(a_{m+1,P} - U\Gamma_j)}\right| \quad \forall\, j \in NB$$

with the arrival Pλ to the base and due to possibility of problem is obvious and by applying equation (3) the benefits are well established. Because Pλ is a positive value in the base, so DMU_P is a efficient DMUP.

4 Numerical examples

Here are 5 examples to expression.

Example 1-4 considers 5 Dmp with two inputs and one output that its coordinates are listed in Table 1:

Table 1: inputs and one output

DMU	1	2	3	4	5
X_{1j}	6	4	2	1	2
X_{2j}	0.5	1	2	4	3
Y_{1j}	1	1	1	1	1

In evaluating the CCR models with covering form, $DMU_1,\dots,DMU_4$, efficient and DMU_5 recognize inefficient. To assess From DMU5 we have:

$\theta_5^*=0.85$, $\lambda^*_3=0.71$, $\lambda^*_4=0.29$

According to Theorem 2, the following new entry index value is calculated for DMU5:

$X_{3,5} \geq 0X_{3,1} + 0X_{3,2} + 0.83 X_{3,3} + 0.33X_{3,4}$

If the input index given in the above equation applies to DMU, the amount of the performance DMU5 keeps constant and equal to 0.85.

Example 2-4 DMU5 information with one input and one output is given in Table 2:

Table 2: DMU5 information with one input and one output

DMU	1	2	3	4	5
X_{IJ}	1	2	3	5	8
Y_{1J}	1	2.5	4	6	2

With evaluating DMU by help of BCC covering form, $DMU_1 \dots DMU_4$ are efficient and DMU_5 is efficient. In evaluating inefficient DMU5we have:

1. $-0.67X_{2,1} - 0.33X_{2,3} + 0.2X_{2,5} < 0$

2. $-x_{2,2} + 0.5x_{2,1} + 0.5x_{2,3} > 0$

3. $\begin{cases} -1.6x_{2,1} - 3.2x_{2,2} + x_{2,5} \geq 0 \\ -6.4x_{2,2} + 1.6x_{2,3} + x_{2,5} \geq 0 \end{cases}$

Desired region to maintain DMU5 performance, in case of adding input index from feed (1), (2) and (3) is obtained.

Example 3-4 Coordinates DMU4 with two inputs and one output according to Table 3, we have:

Table 3: The data relating to 4-unit decision

DMU	1	2	3	4
X_{1J}	1	4	2	5
X_{2J}	5	2.5	3	1
Y_{1J}	1	1	1	1

By solving Form a coating by solving CCR model for each DMU News, DMU1, DMU3, DMU4 is efficient and inefficient DMU2.To determine the value index the output for the inefficient DMU2 case (4) we have:

$\theta_2^*=0.83$ $\lambda^*_3=0.55$ $\lambda^*_4=0.45$

$0 < y_{2,2} \leq 0.55y_{2,3} + 0.45y_{2,4}$

If the output index for DMU applies in relation to high performance levels DMU2 remain constant.

Example 4-4 the coordinates related to DMU5 with two inputs and one output is shown in Table 4:

Table 4: coordinates related to DMU5 with two inputs and one output

DMU	1	2	3	4	5
X_{1J}	1	2	4	6	3
X_{2J}	4	3	1	0.5	2. 5
Y_{1J}	1	1	1	1	1

In evaluating with CCR model in covering form, DMU_1,….,$DMU_4$are efficient and DMU_5 is inefficient.DMU5 to assess the following information is obtained:
$^*\theta=0.9$ $\lambda^*_1=0.42$ $\lambda^*_3=0.58$
Considering that value of the corresponding variable 2λ in goal row is zero in optimal table and issue has other optimal solution.So by establishing the conditions of theorem (5) for the output parameters are:

1 $0.42y_{2,1}+0.58y_{2,3}-y_{2,5}<0$

2 $-y_{2,2}-0.6y_{2,1}-0.3y_{2,3}>0$

3 $\begin{cases}-0.42y_{2,2}+0.22y_{2,3}-0.6_{2,5}\geq 0\\ -0.58y_{2,2}-0.22y_{2,1}-0.3y_{2,5}\geq 0\end{cases}$

District desirable to maintain the performance when adding index entries 5 demo account relations (1), (2) and (3) is obtained.
Example 5-4- DMU3 the specifications listed in Table 5, is given:

DMU	1	2	3
X_{IJ}	3.5	2	4
Y_{1J}	4	2.5	5

By solving the BCC model,DMU1 inefficient and DMU2 ,DMU3 are efficient.
With the conditions of the theorem (6) identified the following desirable features:

$$S=\left\{0.9x_{2,1}-0.4x_{2,2}-0.6x_{2,3}<0, x_{2,1}-0.4x_{2,2}-0.6x_{2,3}<0, 0.22x_{2,2}-0.04x_{2,2}-0.17x_{2,3}<0\right\}$$

For example, with the values of the input parameters as follows, inefficient DMU1 become efficient and maintain performance of DMU3 DMU2.
$X_{2,.1}=0.5$ $x_{2,2}=2$

4. Conclusions

Sensitivity analysis of an issue because of its importance in linear programming problems considered and has been studied in different branches. In this paper, a method was proposed to analyze the sensitivity Stability region so that we are adding a new index to all DMU efficiency units does not change, for creating this area we use the LP sensitivity analysis[11]. To find area ,there is need to solve a linear programming issue, Just a simple calculation can be found in relation to area movement of new input and output so that the efficiency of all units remain constant. It can be used for issue allocation of costs and revenues. Finding area of sustainability in a state of adding, more than one factor could be the subject of further research.

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